Nutrients/food chain model for Lake Zapotlan (Mexico)

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ABSTRACT
A nutrients/food chain model was developed considering the linkage between water balance and energy balance models for Lake Zapotlan. Lake Zapotlan is located in the southern part of Jalisco State, Mexico, and is classified as a warm, tropical water body, endorheic, very shallow, and highly eutrophic. Kinetic parameters were calibrated using previously reported values in the literature for other lakes. The model simulates the concentration of eight state variables for the lake: algae, herbivore and carnivore zooplankton, nitrates, ammonium, SRP, DOC and POC. A set of eight simultaneous ordinary differential equations (ODE) were generated, assuming an initial set of conditions considering the existing hydrologic and water quality information of the streams flowing into the lake. The ODE system was solved and calibrated by using observed water quality data of the lake. In this way, we have estimated the internal fluxes of nutrients and dissolved oxygen in the lake and their interaction with the food chain.

Keywords: Eutrophication; food chain; flux of nutrients; nutrients load; tropical lakes.

1 Introduction

The Lake Zapotlan Basin is located in the southern part of Jalisco State, Mexico, between 19°34' and 19°53' north latitude and 103°24' and 103°38' west longitude. This basin is the second most populous in Jalisco State, with a population close to one hundred thousand inhabitants [25]. This shallow and endorheic lake shows signs of severe eutrophication, manifested by the presence of large stands of aquatic macrophytes (common cattail [Typha latifolia], aquatic hyacinth [Eichhornia crassipes]), and periodic blooms of blue-green algae (Aphanocapsa elachista, Chroococcus limneticus, Dactylococcopsis acicularis, and Microcystis aeruginosa, between others). The main nutrient sources are from continuous discharges of poorly treated sewage, urban runoff from the cities of Ciudad Guzmán and San Sebastián del Sur, and agricultural and farming runoff around the lake basin [36].

Nutrient/food chain models attempt to characterize the partitioning of matter within the lake on a seasonal basis. These models typically have a number of common characteristics, including transport and kinetic characterization. The variables describing the biological processes include several forms of nutrients and a food chain that relates the different interaction between living microorganisms [46, 50]. These models were primarily developed from the 1970’s onwards to address the impact of nutrients on natural waters [4, 9] and include the following nutrients models: CE-QUAL-W2, a water quality and hydrodynamic model in 2D for water bodies basin systems [12]; PCLAKE, a model that calculates the growth of algae, fish and plants in shallow lakes, within the framework of closed nutrient cycles [28, 26]; a coupled watershed and lake model (AnnAGNPS and BATHTUB) that simulates the lake water quality conditions of a reservoir considering changes in different watershed land use and management scenarios [27].

In the last two years several water quality models for lakes and reservoirs have also been developed. Some of these are described below:
(a) A time dependent mathematical model that accounts for the vertical fluxes of nutrients into a lake [23];
(b) a shallow lake model that takes into account the inherent variability in precipitation and uncertainties in the empirical relationships determining nutrient export from stream catchments [38];
(c) LAKELOAD, an integrated model that estimates the loadings of nutrients and organic matter from point and non-point sources in a watershed and accounts for the loadings from industrial and municipal wastewaters, atmospheric deposition and runoff from the watershed [40];
(d) macroscopic food web model that identifies the key and redundant uncertainties associated with the constituent processes of the reservoir ecosystem of Lake Lanier [37];

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(e) a model that applies future scenarios for a lake basin, which are then translated into quantitative changes in the input variables for a GIS system based nutrient transport model [34];

(f) steady state lake phosphorus mass balance model that is used to predict the equilibrium phosphorus concentration of the lake from natural and anthropogenic, external and internal phosphorus inputs [35];

(g) MODIFFUS, a nutrient-flow simulation model that estimates nutrient inputs from diffuse sources into surface waters [39];

(h) a eutrophication model that simulates plankton dynamics in Lake Washington [2];

(i) a phosphorus management model that aids local officials in decision-making processes pertaining to the management of City Park Lake, a shallow, subtropical, urban hypereutrophic lake [44].

However, due to the shallow and morphometric features of the warm tropical and subtropical Mexican natural lakes, most of the water quality models developed in the literature for temperate and deep lakes and reservoirs fail. In this work, we therefore present the first effort to model a Mexican tropical shallow lake located in a closed basin.

2 Methodology

2.1 Water quantity of Lake Zapotlan

A hydrological balance of Lake Zapotlan was developed considering topo-bathymetric information [5], watershed meteorological data [11], and the initial water level of the lake measured in June of 1982 [5]. A simulation program was also developed to predict the main hydrometric components of the lake for the period between June 1982 and December 2003. This program was developed using a discrete simulation language GPSS (General Purpose Simulation System) [33]. The main output variables of the hydrologic balance simulation were inflow (runoff, precipitation, groundwater infiltration, and sewage) and outflow of lake (water extraction and evapotranspiration) [36].

2.2 Temperature of the lake

The daily solar radiation and the theoretical hours of monthly insolation in a cloudless sky over lake were calculated. Effective hours of monthly insolation, air vapour pressure, wind velocity, and air temperature over lake were estimated [11]. We obtained a differential equation for temperature that was numerically integrated by the fourth order Runge-Kutta method with a time increment of one day [8].

2.3 State variables and modeling the dynamic of the process

The state variables of the model are divided into three main groups: food chain (algae, herbivorous zooplankton, and carnivorous zooplankton), non-living organic carbon (POC and DOC), and nutrients (NH$_4$ – N, NO$_3$ – N, and SRP) (See Figure 1). The model simulates the competition between phytoplankton and herbivorous and carnivorous zooplankton. The following assumptions were considered in the simulation model:

(a) algae and detritus are consumed and settle to the bottom of the lake;

(b) algal growth depends on nutrient concentration, solar radiation, and Lake temperature;

(c) total zooplankton biomass increases through the consumption of algae and decreases through excretion, respiration and predation by fishes;

(d) inefficiencies of zooplankton grazing (assumed efficiency of 0.7) provide inputs to the storage of detritus [7];

(e) the mass balances of the food chain and nutrients considered the Lotka-Volterra and Michaelis-Menten equations, respectively [7].

![Figure 1 Nutrient/food-chain model.](image-url)
(f) because of its shallowness and wind action over the surface, the lake was considered as completely mixed horizontally and vertically. Therefore, the main assumption is that a zero-dimensional model could be a good start to describe the chemical and biological processes occurring in the lake.

We have adopted a mechanist, deterministic approach for the model, based on the conservation of mass, and on measured input parameters monitored on a single day during each of the dry and rainy seasons, respectively. Therefore, in this first modeling effort, we did not consider the possible variability in these measurements, and how might this variability affect the conclusions drawn. The equation of the simulation model, in explicit form for every state variable $v$, can be written as follow:

$$\frac{dv}{dt} = \frac{Q_1c_1}{V} + \frac{Q_2c_2}{V} + \frac{Q_3c_3}{V} + \frac{Q_4c_4}{V} - \frac{Q_5c}{V} \pm \frac{FcA}{V \times 1000} + S_v$$

The ratios of chlorophyll-α and carbon and nitrogen/carbon were calculated using the Laws-Chalup model [29] and the solar extinction coefficient was calculated using the Riley equation [43]. Ranges of kinetic parameter values reported in the literature [1, 3, 7, 15, 19, 24, 46, 47, 49] were used to simulate the interaction of nutrients/food chain in Lake Zapotlan. The process velocities $k(T)$ were adjusted by temperature with the theta model [7], shown below,

$$k(T) = k_0 \theta^{T-20}$$

### 3 Estimation of the state variables

#### 3.1 Inorganic nutrients point sources

Table 1 shows the TP and NO$_3$ – N concentrations observed in the sewage discharges to the lake during the dry season (May 8, 2003). In this kind of lake, when measured TP concentrations are higher than 2 mg/L, 80% or more of TP is in the form of SRP [13, 17]. From the hydraulic balance of the water body [36], it was estimated that direct sewage amounted to about 10,300 m$^3$/d discharged into the lake ($Q_1'$), and consequently the mass loads of NO$_3$ – N and SRP were, respectively:

$$W_{n,1} = Q_1' n_{n,1} = 2.88 \times 10^{11} \frac{\mu gN}{d}$$

$$W_{p,1} = Q_1' p_{p,1} = 3.87 \times 10^{11} \frac{\mu gP}{d}$$

Table 1 Nutrient concentrations from direct sewage discharge during the dry season, in mg/L.

<table>
<thead>
<tr>
<th>Sewage discharges</th>
<th>Observed NO$<em>3$ – N ($n</em>{n,1}'$)</th>
<th>Observed TP</th>
<th>Estimated SRP ($p_{p,1}'$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ciudad Guzmán</td>
<td>29.61</td>
<td>47.91</td>
<td>38.33</td>
</tr>
<tr>
<td>San Sebastián del Sur</td>
<td>24.11</td>
<td>46.16</td>
<td>36.93</td>
</tr>
<tr>
<td>Weighted mean</td>
<td>28.00</td>
<td>47.00</td>
<td>37.60</td>
</tr>
</tbody>
</table>

If the mass load velocities remain constant, NO$_3$ – N and SRP concentrations in the sewage discharges can be written as follows, respectively:

$$n_{n,1} = \frac{2.88 \times 10^{11}}{Q_1 \times 1,000}$$

$$p_{p,1} = \frac{3.87 \times 10^{11}}{Q_1 \times 1,000}$$

#### 3.2 DOC point sources

Table 2 shows the estimated DOC in the sewage discharges during the dry season starting from the observed BOD$_5$ [20]:

$$DOC = \frac{BOD_5 + 5.72}{2.06}$$

#### 3.3 Diffuse sources of nutrients

Table 3 shows the diffuse loads of SRP and NO$_3$ – N from the basin, using the export coefficients proposed by Ryding and Rast [45]. The export coefficients for SRP are usually 40 to 50% of that for TP [6]. For the purpose of this calculation, we propose that 50% of the TP load corresponds to SRP. Table 4 shows the diffuse sources of NO$_3$ – N using the export coefficient for tropical watersheds [31]. Generalized export coefficients are used in this paper due to lack of monitoring information related to loadings from individual land uses in the area of study [32] and consequently we did not consider the likely variability in these coefficients, and how might this variability affect the diffuse loads of nutrients from the basin.

Most of the diffuse loads of SRP and NO$_3$ – N reach the lake through surface runoff [42], and their corresponding concentrations are expressed respectively as:

$$p_{n,2} = \frac{W_{n,2}}{Q_2} = \frac{43.94 \times 10^{12} \mu gP}{18.91 \times 10^8 L} = 2,323.67 \frac{\mu gP}{L}$$

$$n_{n,2} = \frac{W_{n,2}}{Q_2} = \frac{103.28 \times 10^{12} \mu gN}{18.91 \times 10^8 L} = 5,461.77 \frac{\mu gN}{L}$$

#### 3.4 Monitoring of water quality

Table 5 shows the TP mean concentration of nine monitoring stations in the lake during the dry and rainy season in 2003. Because we measured only TP, it was assumed that 60% of the observed TP is in the form of SRP, as shown by de Anda et al. [14] in Lake Chapala, which has similar water quality features and is located very close to Lake Zapotlan.

Table 2 BOD$_5$ and DOC concentrations from the sewage (mg/L).

<table>
<thead>
<tr>
<th>Sewage discharges</th>
<th>Observed BOD$_5$</th>
<th>Estimated DOC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ciudad Guzmán</td>
<td>1,181</td>
<td>575</td>
</tr>
<tr>
<td>San Sebastián del Sur</td>
<td>1,363</td>
<td>663</td>
</tr>
<tr>
<td>Weighted concentration</td>
<td>1,272</td>
<td>620</td>
</tr>
</tbody>
</table>
Table 3 Diffuse sources of SRP in the Lake Zapotlan Basin.

<table>
<thead>
<tr>
<th>Diffuse source</th>
<th>Hectares</th>
<th>TP (Kg/ha-yr)</th>
<th>TP (Kg/yr)</th>
<th>SRP (Kg/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Irrigated agriculture</td>
<td>1.576</td>
<td>4.60</td>
<td>7,249.60</td>
<td>3,624.80</td>
</tr>
<tr>
<td>Temporary agriculture</td>
<td>17,425</td>
<td>2.00</td>
<td>3,485.00</td>
<td>17,425.00</td>
</tr>
<tr>
<td>Seeded pasture</td>
<td>2,834</td>
<td>0.10</td>
<td>283.40</td>
<td>141.70</td>
</tr>
<tr>
<td>Coniferous forest</td>
<td>11,960</td>
<td>0.11</td>
<td>1,315.60</td>
<td>657.80</td>
</tr>
<tr>
<td>Deciduous tropical forest</td>
<td>3,278</td>
<td>0.11</td>
<td>360.58</td>
<td>180.29</td>
</tr>
<tr>
<td>Secondary vegetation</td>
<td>3,020</td>
<td>0.10</td>
<td>302.00</td>
<td>151.00</td>
</tr>
<tr>
<td>Urban area</td>
<td>845</td>
<td>1.20</td>
<td>1,014.00</td>
<td>507.00</td>
</tr>
<tr>
<td>Lake and Wetlands</td>
<td>1,562</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Atmospheric deposition over</td>
<td>–</td>
<td>1.00</td>
<td>42,500.00</td>
<td>21,250.00</td>
</tr>
<tr>
<td>the entire basin</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>42,500</td>
<td>87,875.18</td>
<td>43,937.59</td>
<td></td>
</tr>
</tbody>
</table>

Table 4 Diffuse sources of NO\textsubscript{3} - N in the Lake Zapotlan Basin.

<table>
<thead>
<tr>
<th>Diffuse source</th>
<th>Hectares</th>
<th>Exportation rate (Kg/ha-yr)</th>
<th>NO\textsubscript{3} - N (Kg/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basin</td>
<td>42,500</td>
<td>2.43</td>
<td>103,275.00</td>
</tr>
</tbody>
</table>

Table 5 TP and SRP in Lake Zapotlan, in µgP/L.

<table>
<thead>
<tr>
<th></th>
<th>Dry season</th>
<th>Rainy season</th>
</tr>
</thead>
<tbody>
<tr>
<td>(8 May 2003)</td>
<td>(16 September 2003)</td>
<td></td>
</tr>
<tr>
<td>Observed TP</td>
<td>Estimated SRP</td>
<td>Observed TP</td>
</tr>
<tr>
<td>916.00</td>
<td>549.60</td>
<td>1,122.00</td>
</tr>
</tbody>
</table>

The concentrations of NO\textsubscript{3} - N analyzed according to official standard methods (CIATEJ [10]) were low (less than 50.00 µgN/L) during both the dry and rainy seasons. Lewis [30] indicated that the denitrification process can restrict the supply of nitrogen to autotrophic organisms, and can eliminate all the nitrate in the water column in just a few days under anoxic conditions. These conditions could be occurring during the dry season in Lake Zapotlan, and therefore we assumed a NO\textsubscript{3} - N concentration of 0 µg/L during this season. A NO\textsubscript{3} - N concentration of 45 µg/L was assumed for the rainy season because of the load of nitrates to lake via surface. This value does not contradict the reported values by CIATEJ [10], and in this way the average annual NO\textsubscript{3} - N concentration of the lake would be around 23.00 µgN/L, a concentration of NO\textsubscript{3} - N similar to that reported for the neighboring Lake Chapala [21], which has similar water quality of Lake Zapotlan.

The NH\textsubscript{4} - N concentration was estimated from Figure 2 and Table 5. This figure is based on analysis of 450 lakes distributed around the world [41]. From Table 5, the measured TP concentration during the dry season in the lake was 916.00 µgP/L. For this TP concentration, a NO\textsubscript{3} - N : NH\textsubscript{4} - N ratio = 0.73 was provided by Figure 2. As the NO\textsubscript{3} - N concentration of 0 µgN/L was assumed for the dry season, therefore the NH\textsubscript{4} - N concentration is 0 µgN/L. On the other hand, from Table 5, the measured TP concentration during the rainy season in the lake was 1,122.00 µgP/L. For this TP concentration, a NO\textsubscript{3} - N : NH\textsubscript{4} - N ratio = 0.70 was provided by Figure 2. As the NO\textsubscript{3} - N concentration of 45 µgN/L was assumed for the rainy season, therefore the NH\textsubscript{4} - N concentration is 64.30 µgN/L.

3.5 TOC in the lake

Guzmán et al. [22] reported a mean TOC concentration in the lake of 11.50 mgC/L. In the model, we used the assumption DOC/POC = 6 during the simulation period according to Wetzel [52]. Consequently, the DOC and POC mean concentration values can be calculated as 9.90 mgC/L and 1.60 mgC/L, respectively. DOC concentrations in freshwaters are generally in the range 2–10 mgC/L [48].

3.6 Un-ionized ammonia in the lake

Un-ionized ammonia nitrogen concentration \( n_g \) in the lake (in µgN/L) can be calculated as

\[
    n_g = \left( \frac{F_u}{1 - F_u} \right) n_a \tag{10}
\]

where \( F_u \) = fraction of \( n_g \) in total ammonia nitrogen, which depends on temperature \( T(K) \) and pH [18],

\[
    F_u = \frac{1}{1 + 10^{0.9018+0.72992-pH}} \tag{11}
\]

3.7 POM flux in the lake

We have considered that 50% of the BOD\textsubscript{5} in the lake is POM [7]. It is also possible to describe the flux of POM to the sediment (\( J_{CV} \)) by using equivalents of oxygen (gO/m\textsuperscript{2} · d) [7],

\[
    J_{CV} = v_p L_{pw} \tag{12}
\]
3.8 Nutrient flux in the lake

We have assumed that the nutrient concentration \( n (\mu g/L) \) in the lake depends linearly on the surface runoff, according to the equation:

\[
n = n_E + \left( \frac{n_T - n_E}{Q_{2, k}} \right) Q_{2, k} \tag{13}
\]

Flux \( F_n \) of nutrient \( n \) (in \( \mu g/m^2 \cdot d \)) at time \( t \) can be written as:

\[
F_n = \frac{(n - \hat{n})(V \times 1000)}{A \times h} \tag{14}
\]

We suspect that an important part of the flux of nutrients in the lake is eliminated from the water column and consumed by two main components of the ecosystem: aquatic weeds such as common cattail (Typha latifolia) and aquatic hyacinth (Eichhornia crassipes) and the harvest of fishes in the lake, mainly carp and tilapia. Unfortunately, one of the limitations of the proposed model is that it does not take into account a macrophyte and fishery compartment. Future work will further develop the model to take these components into consideration.

3.9 Total oxygen demand in the sediment

Total sediment oxygen demand (TSOD) (carbonaceous and nitrogenous, in gO\(_2\)/m\(^2\) \cdot d) in the bottom sediments of the lake was calculated by applying the model developed by Di Toro et al. [16] that was solved by the secant modified method [8].

3.10 The ordinary differential equations system

The ordinary differential simultaneous equations (ODE) system describing the mass balance of different chemical and biological species describing the lake ecosystem (Table 6) were solved by using the fourth order Runge-Kutta method with a time increment of 0.1 days [8].

3.11 Model calibration

The solution of the ODE-system shown in Table 6 requires eight initial conditions. Through iterative simulations for the period of 2002–2003, we observed a cyclic annual pattern in the values of the state variables. Once we introduced a series of different initial conditions for the first year period, the starting conditions during the second year changed slightly and then remained almost constant. Hence, we established these conditions as the initial ones and the calculation process was performed in an iterative way until the ODE-system was stabilized. Finally, the stabilized initial conditions were considered as the initial conditions for the model. Table 7 shows the initial conditions and assumed nutrient loads to Lake Zapotlan starting on January 1st, 2002.

Nevertheless, it is possible that many different sets of initial conditions may allow the model to fit the observed data equally well yet yield different results in prediction. A numerical factorial experiment \( 2^5 \) was carried out within the model to estimate the susceptibility of the model to such behavior in terms of the RMSE error. With this purpose, the sets of initial conditions of the five state variables of Table 7 (ammonium nitrogen, nitrate nitrogen, SRP, particulate organic carbon, and dissolved organic carbon) were changed. We selected two levels for each initial condition of these mentioned variables. One level was fixed above the value showed in Table 7 and the other was fixed below this value, and consequently \( 2^5 = 32 \) different sets of initial conditions were generated. After that, the model was run for each combination. During the calculation procedure, an RMSE error for each state variable was estimated from 32 combinations. The results and the fixed levels for each initial condition of the selected variables are shown in Table 8.

Once this step was accomplished, the goal of the calibration process was focused on the adjustment of the kinetics parameters until the model outputs reproduced the observed data in the monitoring stations, and also the reported water quality data observed.

<table>
<thead>
<tr>
<th>State variable</th>
<th>Symbol</th>
<th>Mass balance equation</th>
</tr>
</thead>
</table>
| Algae                        | \( a \) | \[
\frac{da}{dt} = \frac{Q_1 a_1}{V} + \frac{Q_2 a_2}{V} - \frac{Q a}{V} + S_a
\] |
| Herbivorous zooplankton      | \( z_h \) | \[
\frac{dz_h}{dt} = \frac{Q_1 z_{h, 1}}{V} + \frac{Q_2 z_{h, 2}}{V} - \frac{Q z_h}{V} + S_{z_h}
\] |
| Carnivorous zooplankton      | \( z_c \) | \[
\frac{dz_c}{dt} = \frac{Q_1 z_{c, 1}}{V} + \frac{Q_2 z_{c, 2}}{V} - \frac{Q z_c}{V} + S_{z_c}
\] |
| Particulate organic carbon   | \( c_p \) | \[
\frac{dc_p}{dt} = \frac{Q_1 c_{p, 1}}{V} + \frac{Q_2 c_{p, 2}}{V} - \frac{Q c_p}{V} + S_{c_p}
\] |
| Dissolved organic carbon     | \( c_d \) | \[
\frac{dc_d}{dt} = \frac{Q_1 c_{d, 1}}{V} + \frac{Q_2 c_{d, 2}}{V} - \frac{Q c_d}{V} + S_{c_d}
\] |
| Ammonium nitrogen            | \( n_a \) | \[
\frac{dn_a}{dt} = \frac{Q_1 n_{a, 1}}{V} + \frac{Q_2 n_{a, 2}}{V} - \frac{Q n_a}{V} \pm \frac{F_{n_a} A}{V \times 1000} + S_{n_a}
\] |
| Nitrate nitrogen             | \( n_i \) | \[
\frac{dn_i}{dt} = \frac{Q_1 n_{i, 1}}{V} + \frac{Q_2 n_{i, 2}}{V} - \frac{Q n_i}{V} \pm \frac{F_{n_i} A}{V \times 1000} + S_{n_i}
\] |
| Soluble reactive phosphorus  | \( p_i \) | \[
\frac{dp_i}{dt} = \frac{Q_1 p_{i, 1}}{V} + \frac{Q_2 p_{i, 2}}{V} + \frac{Q p_i}{V} \pm \frac{F_{p_i} A}{V \times 1000} + S_{p_i}
\] |
4 Results and discussion

The concentrations of the inorganic nutrients in the lake were positively correlated with the lake temperature, lake inflow, pH, DO, TSOD, herbivorous and carnivorous zooplankton concentration, nutrients flux and TOC in the lake, and negatively correlated with the lake outflow and POM flux in the lake ($P < 0.0001$). There was no correlation between herbivorous zooplankton concentrations in the lake with other variables, but there was a high positive correlation between carnivorous zooplankton and lake temperature, photoperiod, lake inflow, pH, DO, TSOD, ammonium flux, nutrient concentration, algal concentrations and TOC ($P < 0.0001$). Algal concentrations were positively correlated with the lake temperature, photoperiod, lake inflow, pH, DO, TSOD, ammonium flux, nutrient concentration, carnivorous zooplankton and TOC concentration; and negatively correlated with lake outflow and POM flux in the lake ($P < 0.0001$). TSOD was positively correlated with the lake temperature, lake inflow, pH, DO, TSOD, ammonium flux, nutrient concentration, carnivorous zooplankton and TOC concentration; and was negatively correlated with the lake outflow and POM flux in the lake ($P < 0.0001$). Lake residence time $\Theta$ was positively correlated with the depth; and negatively correlated with the lake outflow and solar radiation ($P < 0.0001$). From Figure 3 we assumed that nitrogen was the limiting nutrient in the water body, since SRP simulated values were always in excess of total N values during the simulation period. This figure shows that simulated N and P values peaked in July to September, with minima in the winter (December to April).

In terms of nutrient fluxes (Figure 4) we observe a minimum in the fluxes of nitrates, ammonium, and SRP in March to May (spring) with a subsequent peak in fluxes in the summer, corresponding to the peak in nutrient concentrations. The March–April–May minimum in the downward flux of nutrients is due to the intensity of the upward flux of nutrients (mainly caused by resuspension) because of decreased water level in spring and correlated with the monthly average wind speed. During this quarter the strongest winds blow across Lake Zapotlan (see Figure 5). The increase in nutrient concentrations in the lake corresponds to an increase in algal biomass, with a subsequent increase in biomass of herbivorous and carnivorous zooplankton (see Figure 6).

During the rainy season, we observed an increase in the nutrient concentrations in the lake, and consequently pH, DO,
**Figure 4** Simulation of the nutrient fluxes in Lake Zapotlan.

- Nitrate Nitrogen Flux
- SRP Flux
- Ammonium Nitrogen Flux

**Figure 5** Basic atmospheric parameters for Lake Zapotlan.

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar Radiation</td>
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<tr>
<td>Air Temperature</td>
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**Figure 6** Food chain simulation in Lake Zapotlan.

**Figure 7** Organic carbon simulation in Lake Zapotlan.

**TSOD**, flux of nutrients, and **DOC** increased as well. On the other hand, the flux of **POM** to the bottom sediments decreased. Using Eq. (11), we calculated that in the rainy season more than 62% of the total ammonia was in the form of gaseous ammonia, whose maximum concentration exceeded 100 µgN/L, a concentration level that is normally toxic for most fish species. Using the observed monitoring data, the **NO₃-N** concentration in the model was always less than 50 µg/L during the simulated period and the observed **SRP** concentration was adjusted to the model (see Figure 3). During the rainy season there was an increased flux of nutrients from the water column to the bottom sediments (Figure 4). Therefore the simulation shows an increase in the values of **pH**, **DO**, **TSOD**, nutrient concentrations in the water column and **DOC**. During the dry season, the flux of ammonium and **SRP** from the water column to the bottom sediments decreased, and nitrates entered the water column from bottom sediments.

Figure 6 shows the results of the food chain simulation in the lake. Zooplankton concentrations were expressed in chlorophyll-α concentration units in order to compare them to the behavior of algal concentrations [7]. Results show strong interaction between predators and prey. Algae showed a rapid increase in concentration during the second quarter of the year, and then decreased gradually until December. The maximum herbivorous zooplankton concentration level occurred in May and the minimum occurred in February.

Figure 7 shows the difference in the productivity between the rainy season and the dry season. High organic carbon generation during the rainy season has several implications. Its decomposition can have an impact on the oxygen content of bottom waters and the aquatic biota. Besides, the transport and fate of substances such as heavy metals can be strongly associated with organic matter [7]. At the same time that organic carbon generation increases, the flux of **POM** to the bottom sediments decreases, while **pH**, **DO**, **TSOD**, the flux of ammonium from the water column, nutrients, herbivorous and carnivorous zooplankton concentrations in the lake increase. As a result of the simulation, we obtained a **DOC**/**POC** rate of 6 [52] and the same **TOC** mean concentration reported by Guzmán et al. [22]. Figure 8 shows the **TSOD** and the flux of **POM** to the bottom sediments of the lake. Maximum of **TSOD** occurred during the rainy season, when minimum fluxes of **POM** to the bottom sediments occurred.
surrounding the lake could involve the creation of a vegetated, non-cultivated riparian zone around the lake, acting as a buffer to capture the excess of nutrients. Presently there is no system protecting the shoreline of the lake from the agrochemicals used in local agriculture.

The proposed model integrates the hydrologic balance, the energy equations, and the food chain processes in the lake. Therefore, it is possible to simulate different scenarios to improve the water quality of the system by reducing the loads of nutrients. This tool permits decision makers to focus administrative and economic resources on those measures that give better results in terms of water quantity and quality in the lake.

The next step in the development of this model will be to include a fisheries and aquatic macrophytes component, as noted earlier. Another of the areas of concern in this lake is the existence of blue-green algae blooms producing cyanotoxins. This model will be improved to determine the processes that permit the development of such algae.

\section{Conclusions}

Lake Zapotlan is a dynamic ecosystem, dominated by the interactions amongst solar radiation, surface runoff and discharge of wastewater. In early spring, when the wind restricts the flux of SRP and ammonia to the bottom and mixes nitrate throughout the water column, we have observed an increase in primary productivity in the lake. The calculated increase in primary productivity is attributable to the diffuse loading of nutrients mainly during the rainy season, and to the slow increase of the air temperature, nitrification, hydrolysis of DOC, carnivorous zooplankton respiration, etc. Primary productivity decreases at the beginning of the summer because of the following factors:

(a) Increase in the ammonium flux to the bottom sediments;
(b) diminution of the nitrate flux from the bottom sediments to the water column;
(c) increase of the POM flux to the bottom sediments; and the
(d) limited amount of TOC present in the water column.

The lowest levels of nutrients and primary production occur in winter and at the beginning of spring, principally due to low levels of DOC in the lake, which results in a low generation of nutrients via hydrolysis.

The simplified nutrients/food chain dynamic model developed in this work permits a better understanding of the processes occurring in the lake. It describes the impact of the point and non-point sources of nutrients on the water quality of the lake and the growth and composition of the food chain. The main point source of nutrients affecting the water quality in the system is the direct discharge of poorly treated municipal sewage into the lake. It is necessary to improve sewage treatment by removing nutrients prior to its discharge into the lake. Alternatively one could consider diversion of treated sewage to cultivated fields (reuse of water). This measure would also minimize the extraction of groundwater from wells. The control of the diffuse source of nutrients is a complicated issue that requires the development of a management plan for the entire lake basin. A rapid action plan that minimizes the load of nutrients from agricultural areas

\section{Notation}

\begin{itemize}
  \item $a =$ Concentration of algae in the Lake ($\mu g$/chl/L)
  \item $A =$ Area of the sediment-water interface $\approx$ Lake area ($m^2$)
  \item $a_{ij} =$ Concentration of algae in the flow $j$($\mu g$/chl/L)
  \item $BOD_5 =$ 5-day biochemical oxygen demand
  \item $c =$ Concentration of state variable $v$ in the lake ($\mu g$/m$^3$ or mg/m$^3$)
  \item $c_{ij} =$ Concentration of state variable $v$ in the flow $i$ ($\mu g$/m$^3$ or mg/m$^3$)
  \item $c_d =$ Concentration of DOC in the Lake (mgC/L)
  \item $c_{dj} =$ Concentration of DOC in the flow $j$ (mgC/L)
  \item $c_p =$ Concentration of POC in the Lake (mgC/L)
  \item $c_{p,j} =$ Concentration of POC in the flow $j$ (mgC/L)
  \item $DOC =$ Dissolved organic carbon
  \item $f =$ Photoperiod
  \item $F_n =$ Flux of nutrient $n$ at time $t$ (mg/m$^2$ \cdot d)
  \item $F_v =$ Flux of the state variable $v$ in the interface (when nutrient) ($\mu g$/m$^2$ \cdot d) (A positive sign denotes a flux directed towards the water column, a negative sign denotes an opposite flux).
  \item $h =$ Time increment (days)
  \item $I_d =$ Solar radiation at the lake surface
  \item $j =$ Flow $= 1$(sewage), $2$(runoff), $3$(surface infiltration), $4$(precipitation), $5$(water extraction)
  \item $J_{CN} =$ flux of POM to the sediment (gO/m$^2$ \cdot d)
  \item $k(T) =$ process velocity at temperature $T$
  \item $k_{20} =$ process velocity at 20°C
  \item $L_{POM} =$ POM concentration in the lake in the time $t$ (mgO/L)
  \item $n_w =$ Concentration of NH$_4$ $- N$ in the Lake ($\mu g$/N/L)
  \item $n_{w,j} =$ Concentration of NH$_4$ $- N$ in the flow $j$ ($\mu g$/N/L)
  \item $n_e =$ Concentration during the dry season ($\mu g$/L)
  \item $n_i =$ Concentration of NH$_4$ $- N$ in the Lake ($\mu g$/N/L)\end{itemize}
\( n_g \) = Concentration of un-ionized \( NH_3 - N \) in the Lake (\( \mu g/N/L \))

\( n'_{i,1} \) = Concentration of \( NO_3 - N \) at the sewage on May 8, 2003 (\( \mu g/N/L \))

\( n_{i,j} \) = Concentration of \( NH_4 - N \) in the flow \( j(\mu g/N/L) \)

\( n_T \) = Concentration in the month of maximum surface runoff (\( \mu g/L \))

\( \dot{n} \) = Concentration of nutrient \( n \) calculated by the model without considering any flux (\( \mu g/L \))

\( DO \) = Concentration of oxygen dissolved in the Lake (\( \mu g/L \))

\( POC \) = Particulate organic carbon

\( POM \) = Particulate organic matter

\( p_s \) = Concentration of SRP in the Lake (\( \mu g/P/L \))

\( p_{s,j} \) = Concentration of SRP in the flow \( j(\mu g/P/L) \)

\( p'_{s,1} \) = Concentration of SRP at the sewage on May 8, 2003 (\( \mu g/P/L \))

\( Q_j \) = Volumetric flow rate of the flow \( j(\text{m}^3/\text{d}) \)

\( Q_{2,k} \) = Surface runoff in the month \( k(\text{m}^3/\text{month}) \)

\( Q_{2,T} \) = Maximum surface runoff during the rainy season (\( \text{m}^3/\text{month} \))

\( Q_2' \) = Annual average of surface runoff (L)

\( Q_1' \) = Sewage discharges to the lake on May 8, 2003 (\( \text{m}^3/\text{d} \))

\( S_s \) = Sources and sinks of the state variable

\( \nu(\mu g/L \cdot \text{d}) \), calculated by the Chapra model [7].

\( RMSE \) = Root mean square error

\( SRP \) = Soluble reactive phosphorus

\( t \) = Time (d)

\( T \) = Lake temperature (°C)

\( TOC \) = Total organic carbon

\( TP \) = Total phosphorus

\( TSOD \) = Total oxygen demand in the bottom sediments of the lake (\( \text{gO/m}^2 \cdot \text{d} \))

\( v \) = State variable = \( a, z_h, z_c, c_p, c_d, n_a, p_s \) or \( n_i \)

\( V \) = Lake volume (\( \text{m}^3 \))

\( v_p \) = \( POM \) settling velocity = 0.2 m/d [7]

\( W_{p,1} \) = Mass loads of \( NO_3 - N \) from point sources on May 8, 2003 (\( \mu g \))

\( W_{p,3} \) = Mass load of SRP from point sources on May 8, 2003 (\( \mu g \))

\( W_{n,1} \) = Annual diffuse loads of \( NO_3 - N \) from the basin (\( \mu g \))

\( W_{p,i} \) = Annual diffuse loads of SRP from the basin (\( \mu g \))

\( z_c \) = Concentration of carnivorous zooplankton in the Lake (\( \text{mgC/L} \))

\( z_c,j \) = Concentration of carnivorous zooplankton in the flow \( j(\text{mgC/L}) \)

\( z_h \) = Concentration of herbivorous zooplankton in the Lake (\( \text{mgC/L} \))

\( z_h,j \) = Concentration of herbivorous zooplankton in the flow \( j(\text{mgC/L}) \)

\( \theta \) = Temperature factor

\( \Theta \) = Lake residence time (months)

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